Sea level variations during snowball Earth formation: 1. A preliminary analysis

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[1] A preliminary theoretical estimate of the extent to which the ocean surface could have fallen with respect to the continents during the snowball Earth events of the Late Neoproterozoic is made by solving the Sea Level Equation for a spherically symmetric Maxwell Earth. For a 720 Ma (Sturtian) continental configuration, the ice sheet volume in a snowball state is ~750 m sea level equivalent, but ocean surface lowering (relative to the original surface) is ~525 m due to ocean floor rebounding. Because the land is depressed by ice sheets nonuniformly, the continental freeboard (which may be recorded in the sedimentary record) at the edge of the continents varies between 280 and 520 m. For the 570 Ma (Marinoan) continental configuration, ice volumes are ~1013 m in eustatic sea level equivalent in a "soft snowball" event and ~1047 m in a "hard snowball" event. For this more recent of the two major Neoproterozoic glaciations, the inferred freeboard generally ranges from 530 to 890 m with most probable values around 620 m. The thickness of the elastic lithosphere has more influence on the predicted freeboard values than does the viscosity of the mantle, but the influence is still small (~20 m). We therefore find that the expected continental freeboard during a snowball Earth event is broadly consistent with expectations (~500 m) based upon the inferences from Otavi Group sediments.

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1. Introduction

[2] The existence of low paleo-latitude glacial deposits overlain by thick successions of "cap carbonates" and the associated dramatic changes in the carbon isotopic ratio δ^{13}C, which is proxy for photosynthetic activity, provide convincing evidence that a series of global glaciation events may have occurred during the Neoproterozoic (1000 – 540 Ma) interval of geological time [see reviews by *Hoffman and Schrag*, 2000, 2002; *Hoffman and Li*, 2009]. These events have been termed "snowball Earth" events by *Kirschvink* [1992]. Due to the continuous recycling of ocean floor back into the mantle in association with the mantle convection process, any evidence directly indicative of the extent of sea ice change during these events has long since disappeared. This has led to the sharpening of the debate between the proponents of two end-member scenarios for the Earth's surface during these events, i.e., the "hard snowball" end-member and the "soft snowball/slushball" end-member. The former hypothesis proposes that the ocean was completely covered by a thick (100 s of meters thick) veneer of sea ice [e.g., *Hoffman et al*., 1998], while the latter advocates that the tropical oceans were either free of ice [e.g., *Hyde et al*., 2000] or had very thin (10 s of meters thick) sea ice [*Pollard and Kasting*, 2005] so that sunlight could still penetrate into the ocean, thereby allowing photosynthetic activity to continue. The advocates of each of these views have also proposed mechanisms to eventually melt the glaciated surface and to explain the carbon isotopic ratio variations [*Kirschvink*, 1992; *Peltier et al*., 2007; *Liu and Peltier*, 2011; *Johnston et al*., 2012]. While this debate is still ongoing, our goal in this paper is to attempt to resolve an additional puzzle, namely that concerning the sea level change that is expected to have occurred during the development of a snowball Earth state, whether "hard" or "soft."

[3] Whichever of these versions of the snowball hypothesis is correct, both require that most of the continental surface was covered by land ice during the snowball state, and therefore significant falls of mean (average over the whole ocean) sea level are expected. Direct estimates of the accompanying land ice volume variations, based upon the application of a state-of-the-art model of continental scale snowball glaciation events, were recently provided in *Liu and Peltier* [2010] whose analyses delivered ice volumes equivalent to an approximately 1000 m change of eustatic sea level (mean ocean depth) for the Marinoan glaciation and ~800 m for the Sturtian glaciation. These ice volume-based estimates must be seen as crude, however, for at least three reasons: (1) the continental configurations analyzed were unavoidably uncertain, and therefore both the land and ocean areas were

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uncertain; (2) the surface topography of the continents was unknown, and a flat topography for each of the continental fragments in the paleoreconstructions analyzed was assumed; (3) the parameterizations employed in the model for unresolvable processes, especially for precipitation and for the ice sheet flow enhancement factor due to fabric anisotropy and impurities in the ice, were poorly constrained. Nonetheless, the calculations in *Liu and Peltier* [2010] using different paleomagnetism-based continental configurations and precipitation rates all produced ice volumes consistent with an approximately 1000 m change/reduction in mean ocean depth. Such a large change of sea level during snowball Earth events should certainly have been recorded geologically if it was ubiquitous and be observed in appropriate sedimentary stratigraphies that have since been uplifted onto the continents by tectonic processes.

[4] There does, in fact, exist evidence of a snowball Earth-related change in sea level of ~500 m for the Marinoan glaciation from the Otavi Group sediment of Northern Namibia, as discussed in *Hoffman et al*. [2007]. Even if this interpretation of the geological record were correct (it was considered by *Hoffman et al*. [2007] to be a conservative estimate of the associated sea level fall), this is still significantly less than the ~1000 m sea level change expected for a snowball Earth event as analyzed in *Liu and Peltier* [2010] when the estimate is based solely upon the volume of continental ice emplaced during the event. The only other candidate evidence of a large sea level change during the snowball Earth era is probably that based on interpretations of the Wonoka canyons, several of which are more than 1000 m deep [*Christie-Blick et al*., 1990] and could possibly have been produced by the downcutting of rivers flowing to a sea that lay a full kilometer below the surface of the continental catchment. However, the origin of these canyons continues to be a subject of debate [*Christie-Blick et al*., 2001], and more recently, they have been argued by some [e.g., *Giddings et al*., 2010] to be of submarine origin and thus may not be indicating extreme (~1000 m) sea level fluctuations. The contrast between the sparse observational constraints that do exist and expectations based upon the snowball Earth hypothesis therefore suggests the importance of a quantitative analysis of these qualitative expectations.

[5] An important aspect of such a quantitative analysis, a derivative of work that has been performed set out to investigate the relative sea level (RSL) changes that are observed to have accompanied the glaciation and deglaciation events of the Late Quaternary period, involves the application of the Sea Level Equation (SLE) formalism (e.g., see *Peltier* [1998, 2007] for recent reviews). Such analyses have clearly demonstrated that the sea level changes associated with the growth and decay of land ice sheets are highly nonuniform (e.g., see *Peltier* [2004] for analyses based upon the ICE-5G (VM2) model of the global process of glacial isostatic adjustment). The relative sea level (RSL) change that occurs during a continental glaciation event is a complex process involving both the radial displacement (isostatic adjustment) of the surface of the solid Earth and the redistribution of water in the ocean basins that is required to ensure that the surface of the ocean lies on an equipotential surface (over timescales that are much longer than those of the ocean currents involved in redistributing and mixing the seawater of high or low salinity throughout the ocean basins through which this quasi-static state is achieved). For example, during the glaciation of the continents, the mass of ice added to the continent(s) attracts ocean water toward it thus causing a tendency for sea level to rise in the vicinity of the ice-loaded continent, which to a significant degree may compensate or even exceed the general fall of sea level caused by the withdrawal of water from the oceans required to build the continental ice sheet(s) (e.g., see *Peltier* [2007] for a recent review). However, as will be described in detail later, for long timescale events such as that associated with snowball Earth formation, the whole Earth system is able to achieve an equilibrium state in which the difference in relative sea level between the near field (close to the ice sheet) and far field (distant from the ice sheet) is negligible.

[6] In the following, we will present a preliminary study of the magnitude and spatial pattern of the sea level change to be expected during snowball Earth glaciation events, with the primary focus on the coastlines of the glaciated supercontinent. The analyses presented herein should be seen as preliminary because several assumptions/simplifications will be invoked concerning the ice sheet loading of the continents and the topography of the continental surfaces upon which the ice is accumulating. The most important of these simplifications is that the continental ice sheet(s) will be assumed to achieve their maximum extent instantaneously (i.e., the snowball state [see, e.g., of *Liu and Peltier*, 2010, Figure 9]). This simplification is of no consequence if we are interested only in the equilibrium state of relative sea level once the ice mass(es) are fully developed. Furthermore, it is entirely possible that the final phase of significant land ice expansion might have been achieved within a very short time period (Figure 1). The second major simplification we will invoke for the purpose of this preliminary study is that the continents will be assumed to be bounded by "shelves" that are of vanishingly small width so that their coastlines are cliffs beyond which the oceans are deep relative to the changes in sea level associated with the glaciation process. This simplification is expected to be approximately valid because it can be reasonably assumed that the ice sheets that form during a snowball Earth event extended to the edges of any previously existing continental shelves. The third simplification concerns the nature of the ice sheet model that is employed to obtain the land ice distribution in the snowball state. The model we will be employing is the University of Toronto Glacial Systems Model (UofT GSM) that has been developed in the work of *Deblonde and Peltier* [1990, 1993], *Deblonde et al*. [1992], and *Tarasov and Peltier* [1997, 1999, 2003]. This is the same model as was employed in the recent papers of *Peltier et al*. [2007] and *Liu and Peltier* [2010, 2011]. The final assumption that we will make for the purpose of these preliminary analyses concerns the influence of rotational feedback (e.g., see *Peltier* [2007]; and *Peltier et al*. [2012] for recent reviews). This process will be neglected entirely for the purpose of these initial analyses, not because we consider it to be necessarily irrelevant, but because we wish to first establish a baseline of results against which the influence of rotational feedback may be fully understood. In particular, the emplacement of a massive accumulation of continental ice at the onset of a snowball Earth event might be expected to elicit an episode of true polar wander during which the ice-Earth-ocean system could evolve into a new state in which, once again, it spins about its axis of greatest moment of inertia. This could involve

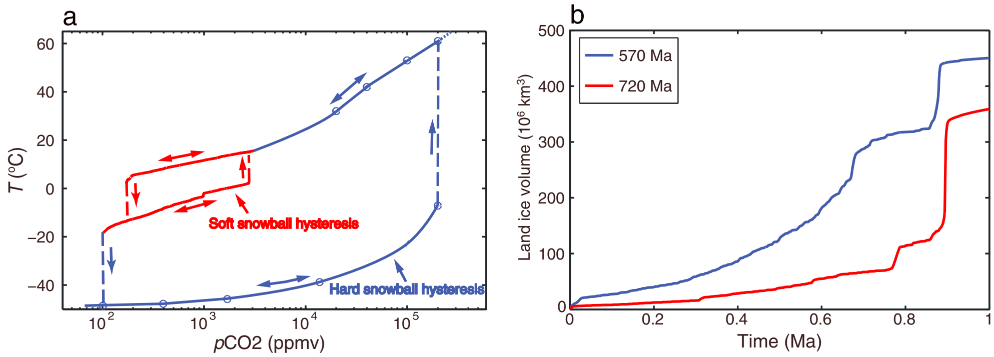

Figure 1. (a) Hysteresis associated with the formation and melting of a soft snowball Earth (red curves) and a hard snowball Earth (blue curves) demonstrated in the T (annually average global mean surface air temperature) - $p\mathrm{CO}_2$ (atmospheric carbon dioxide concentration) space. Solar luminosity is 6% weaker than today. The red curves are modified from Figure 5e of *Liu and Peltier* [2010]; the left vertical blue curve is estimated from *Yang et al*. [2012]; the low horizontal and right vertical blue lines are modified from *Hu et al*. [2011]. Vertical dashed lines (both red and blue) represent occurrence of instabilities (i.e., no steady climate states exist along those lines). Arrows indicate the direction(s) along which climate may evolve. (b) Evolution of ice volume with time during the formation of a soft snowball Earth (toward left along the upper and left branch of the hysteresis in red in Figure 1a) for 570 Ma and 720 Ma continental configurations, respectively. The figure should not be taken to be representative of the actual time it takes for a soft snowball Earth to form because it depends on the rate at which CO_2 concentration is decreased (more details can be found in *Peltier et al*. [2007] and *Liu and Peltier* [2011], who coupled a carbon cycle model to the coupled ice sheet model and energy balance model), but may be taken to be representative of the evolution of land ice volume over time.

a secular effect able to induce a continuous time dependence of sea level history that will cease only when the new stable state of rotation has been achieved. This additional influence upon Neoproterozoic sea level history during a snowball Earth event will be discussed in detail in *Liu and Peltier* [2013].

[7] The goal of our analyses is to provide the first quantitative estimate of the global fingerprint of relative sea level change that is expected to accompany a snowball Earth event. In the first stage of this analysis, we will begin by discussing the snowball glaciation of a single geometrically simple supercontinent to demonstrate the dependence of isostatic adjustment and sea level change upon the horizontal scale of the ice sheets. The ideal continent will be assumed to be circular in shape, and the ice sheet that develops on it to has a cylindrical form with its center coincident with that of the model supercontinent. In a single variant upon this geometry, the ice sheet that develops upon the circular supercontinent will be computed using the detailed glaciological model. These analyses of geometrically simplified models of ice-loaded supercontinents will be followed by the discussion of the results obtained for the fully realistic continental configurations employed in *Liu and Peltier* [2010, 2011].

2. Theoretical Preliminaries: Design of the Numerical Simulations

2.1. Isostatic Adjustment and Sea Level Change Models

[8] Our interest is in changes of relative sea level (RSL) as previously defined, namely the level of the surface of the sea relative to the surface of the solid Earth. This is distinct from what is often referred to as "absolute" sea level which is the level of the sea measured with respect to the center of mass of the planet as a whole, which is most often referred to as the geoid of classical geodesy. Moreover, in the real ocean during the snowball Earth period, sea ice will also be present. Although the formation of sea ice does not modify the geoid as no variation of mass is involved, it does change the position of the physical ocean surface which clearly lies at the base of the floating sea ice. As this is of no consequence in the present context, it will be ignored. When we refer to the surface of the sea in all that follows, we will imply the position of the surface that would exist if all sea ice were melted in regions where it is predicted to exist by the climate model. To facilitate understanding of the discussion to follow concerning the various assumptions involved in these analyses, the generic form of the Sea Level Equation is presented below. As stated above, relative sea level is defined as the difference in elevation between the geoid and the surface of the solid Earth, thus:

$$S(\lambda,\theta,t) = C(\lambda,\theta,t)[G(\lambda,\theta,t) - R(\lambda,\theta,t)] \tag{1}$$

where S, G, and R are the relative sea level, geoid height, and radius of the solid Earth at longitude λ and latitude θ at time t, respectively. C is the ocean function, which is unity where there is ocean and 0 otherwise. All the four variables S, G, R, and C change with time due to the changes of the response to surface loading and the accompanying perturbations to the rotation of the Earth, impacts that may be related as follows:

$$\begin{aligned}\delta S(\lambda,\theta,t) = {} & C(\lambda,\theta,t)[\delta G^L(\lambda,\theta,t) - \delta R^L(\lambda,\theta,t) \\ & + \delta G^T(\lambda,\theta,t) - \delta R^T(\lambda,\theta,t)] + \delta C(\lambda,\theta,t)[G^L(\lambda,\theta,t) \\ & - R^L(\lambda,\theta,t) + G^T(\lambda,\theta,t) - R^T(\lambda,\theta,t)]\end{aligned} \tag{2}$$

where the superscripts "L" and "T" represent surface mass loading-induced and tidal loading (due to change of rotational

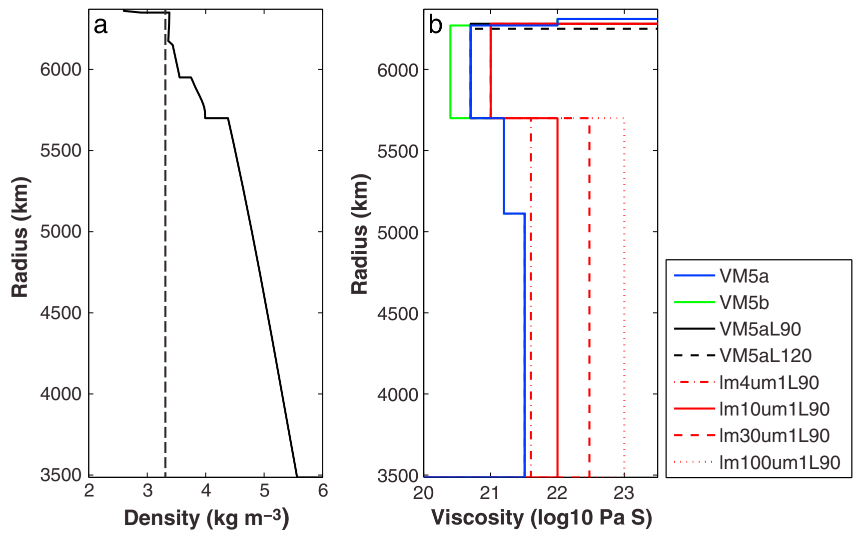

Figure 2. Radial profile of density (PREM [*Dziewonski and Anderson*, 1981]) and viscosity of the mantle where VM5a (blue curve) is from ICE6G_VM5a [*Peltier and Drummond*, 2008]. The dashed line in Figure 2a is a simplified density profile used by model expo_eust only. Please see text for more detailed description of the viscosity models.

state of Earth) -induced deflections of the geoid and surface of the solid Earth, respectively. Since we will be ignoring the influence of rotational feedback in Part 1 of this study, the tidally induced changes in equation (2) will be neglected. Moreover, due the steep continent boundaries employed in this paper, the variation of the ocean function of $\delta C(\lambda,\theta,t)$ vanishes, making the second term of the right-hand side of equation (2) negligible although this term is always evaluated in the software we are employing. Note that both the variations in geoid height and the local radius of the solid Earth are defined and must be solved for over the whole globe. The input to the Sea Level Equation consists of an assumed glaciation history of the continents that is delivered by an ice sheet coupled climate model, and the output from the solution of which provides the associated history of relative sea level change. Both the loading of the continental surface by ice mass and the complementary unloading of the ocean basins are included in the solution of the ice-Earth-ocean interaction problem provided by the Sea Level Equation.

[9] Three progressively more advanced combinations of isostatic adjustment model (solving for R) and relative sea level change model (solving for S) are to be compared in the following. In the first case, the displacement of the bedrock is calculated using the simple exponential return to equilibrium relationship [*Liu and Peltier*, 2010, equation (7)] between the surface mass load and the solid Earth displacement that it induces; in this first case, the sea level change will be approximated by the eustatic change (average over the surface area of the oceans). The displacement of the bedrock is determined only by the ice loading (or water loading if in the ocean basins) change directly above and the difference in density between ice (or water) and bedrock. The characteristic exponential relaxation time to be employed will be 4000 years (τ in equation (7) of *Liu and Peltier* [2010]), which is approximately characteristic of the isostatic adjustment process induced by the removal of the ice sheets of the most recent cycle of glaciation and deglaciation of the Late Quaternary period. This first combination of model characteristics will be referred to as "expo_eust." This methodology has been quite commonly employed (e.g., in the Glimmer ice sheet model by *Rutt et al*. [2009], and in the earliest version of the UofT GSM [*Deblonde and Peltier*, 1990]).

[10] In the second stage of analysis, the sea level change will still be approximated as eustatic, but the bedrock deformation will be calculated using the full isostatic adjustment theory that has been developed by *Peltier* [1974, 1976] (see *Peltier* [1998] for a review). Although the new ocean surface can be obtained by simply lowering the initial surface uniformly by a value determined by the volume of land ice, the determination of the ocean floor displacement at each time step requires a few iterations because as it deforms, the loading/height of the water column above it also changes. This methodology will be referred to as "full_eust." A similar method has also been employed previously in calculating the relative sea level response to Greenland's glaciation history [*Tarasov and Peltier*, 2002].

[11] The third variant upon the methodology to be employed improves upon the way in which relative sea level is calculated compared to the previous two methodologies. In this methodology, the Sea Level Equation (2) is solved in a gravitationally self-consistent fashion using the theory of viscoelastic normal modes, as for example in *Peltier* [2007], but without considering rotational feedback. This method will be referred to as "full_full." The influence of rotational feedback was rarely considered prior to the papers of *Sabadini and Peltier* [1981], *Peltier* [1982], and *Wu and Peltier* [1984]. In the recent literature, however, it has been suggested to be important in enabling the theory to fit Holocene observations of relative sea level history in several distinct geographical regions [*Peltier*, 2002, 2004, 2009] and to assess the present-day ongoing sea level rise [*Peltier et al*., 2012]. The rotational feedback effect will be considered in detail in part 2 of this study as discussed above.

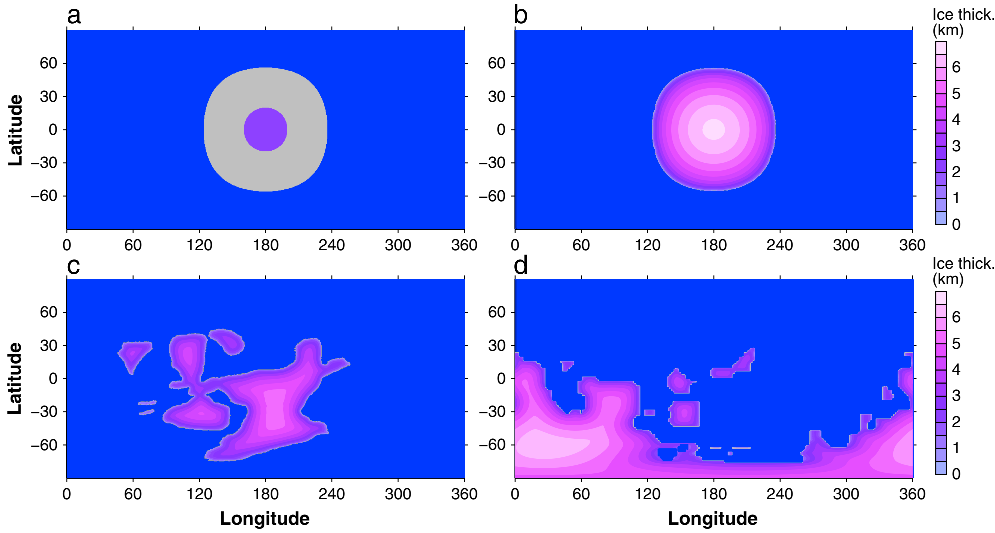

Figure 3. Continental configuration and ice sheet viewed from above. (a) Circular continent (gray area) with cylindrical ice sheet, the radius of which is 20 degrees (mauve contour). (b) Circular continent (fully covered by ice) with the ice sheet volume appropriate to that for the hard snowball state. (c) Continental configuration and ice sheets for 720 Ma. (d) Continental configuration and ice sheets for 570 Ma.

2.2. The Model of Viscoelastic Earth Structure

[12] The Earth is assumed to be both compressible and spherically symmetric, and the radial structures of density (Figure 2a) and elastic properties are taken to be those of PREM [*Dziewonski and Anderson*, 1981]. An exception to this is in the case of the expo_eust methodology in that this method does not require a complete profile of density (solid line in Figure 2a) for the Earth, but rather a single density that is representative of the bedrock. A value of 3300 kg m^{-3} will be employed at this level of approximation, which is the density of typical rocks of the near surface lithosphere (see dashed line in Figure 2a). Eight different radial profiles of viscosity are employed (Figure 2b), including the VM5a model [*Peltier and Drummond*, 2008] which will be employed as the standard model herein. The lithosphere in VM5a has a 60 km thick upper part which is essentially elastic (represented by very high viscosity) and a 40 km thick lower part which has lower viscosity (10^{22} Pa s) but still much higher than that of the upper mantle which has a viscosity of 5×10^{20} Pa s. The lower mantle of this standard model is divided into two layers with the upper 588 km having relatively low viscosity (1.57×10^{21} Pa s) and the lower layer has relatively high viscosity (3.23×10^{21} Pa s). The remaining seven models are VM5b, VM5aL90, VM5aL120, lm4um1L90, lm10um1L90, lm30um1L90, and lm100um1L90. Among them, the three VM5 variations have the same lower mantle (both layers) viscosity as that of VM5a, but the upper mantle viscosity is halved in VM5b; the elastic lithosphere is increased to 90 km and 120 km in VM5aL90 and VM5aL120, respectively, with no low viscosity stratification of the lithosphere. The other four viscosity models, lm4um1L90, lm10um1L90, lm30um1L90, and lm100um1L90, have only three layers from the surface to the core-mantle boundary, a 90 km thick elastic lithosphere, and only the viscosity of the lower mantle differs from that of VM5a, having values of 4×, 10×, 30×, and 100×10^{21} Pa s, respectively. The viscosity of the upper mantle in each of these models is fixed to the value 1×10^{21} Pa s in all four models. The Earth's core is assumed to be inviscid in all eight models.

2.3. Continental Configurations and Model Ice Sheets

[13] In the results section of this paper, we will begin by discussing the ideal supercontinental configuration which is circular in shape (Figures 3a and 3b) and centered on the equator (180°E, 0°N). The radius of the supercontinent will be fixed to 56 degrees, which was chosen so that the area of the continent is similar to that of the elliptical supercontinent discussed in *Liu and Peltier* [2010]. The circular shape was chosen solely for its symmetry, which makes the comparison of the sea level results obtained by the application of the different methodologies simpler to display and therefore easier to understand. More realistic continental configurations for 720 Ma (Figure 3c) and 570 Ma (Figure 3d) will then be discussed. These more realistic configurations were originally reconstructed by *Li et al.* [2008] and *Dalziel* [1997], respectively, but with a slight modification of the former as previously introduced in *Liu and Peltier* [2010, 2011]. Prior to the ice sheet loading of the continents, it will be assumed that the solid Earth is in equilibrium with its ocean load. The continents will initially be assumed to be free of internal topographic relief, with a freeboard of only 10 m (i.e., the surface of the land is everywhere10 m above the initial ocean surface which is the reference (geoidal) surface). The ocean bottom will also be assumed to be flat and to lie uniformly 4000 m below the reference surface.

[14] For the circular continent, two types of ice sheets will be employed and the responses to the associated mass loads analyzed, one of which has an unrealistic cylindrical shape but which will greatly facilitate the analyses, the other of which is realistically calculated using the ice sheet model coupled with an energy balance climate model (EBM) as employed in *Liu and Peltier* [2010]. The height of the cylindrical ice sheet will be fixed to 2000 m, but the radius will be

varied through the sequence 5, 10, 20, 40, and 56 degrees. An example for the radius of 20 degrees is shown in Figure 3a. The realistically shaped ice sheet is obtained by fixing the atmospheric CO_2 concentration to a very low value in the *Peltier et al.* [2007] model so that a hard snowball Earth forms (Figure 3b). For the more realistic continental configurations, the ice sheets are always those calculated by the ice sheet model coupled to the EBM. Their forms in a hard snowball state are shown in Figures 3c and 3d for the 720 Ma and 570 Ma continental configurations, respectively. For the 570 Ma continental configuration, the ice sheet in the soft snowball state is the same as that in Figure 5c of *Peltier et al.* [2007]. For the 720 Ma continental configuration, the ice sheets do not differ significantly between the soft and hard snowball states; therefore, only that for the hard snowball state is analyzed.

[15] In all of the simulations that we will describe, the ice sheets, whether ideal or more realistic, are added to the continents instantaneously at the end of the first time step, and the equivalent mass of sea water is removed from the ocean. This is clearly somewhat unrealistic. Both on the basis of our own experience (Figures 1) and the analyses of others [e.g., *Donnadieu et al.*, 2003], the shift of the climate from a relatively warm state to a (soft) snowball Earth state might take a few hundred thousand years (kyr). Likewise, as will be demonstrated below, sea level achieves a close to equilibrium distribution after a similar period of time. The results to be presented in the following will provide insight into the sea level variations to be expected during the period of the existence of the snowball state as each snowball Earth event is expected to have persisted for millions of years [*Hoffman et al.*, 1998].

[16] The maximum degree and order of the spherical harmonics employed in constructing our solutions to the Sea Level Equation using the pseudo-spectral methodology of *Mitrovica and Peltier* [1991] (note that an error in this paper concerning the density variable required correction) is 256 for the ideal continents and ice sheets unless otherwise specified, and 512 for the more realistic continental configurations and ice sheets. Time steps ranging from 50 years to 10 thousand years (kyr) have been employed depending on how much detail concerning the time evolution of the bedrock deformation or sea level change is required. Large time steps can be employed for our purposes because of the linearity of the problem and the simplicity of the land-ocean configuration (e.g., the assumed cliff-like continent-ocean interface).

3. Results and Discussion

[17] The elevation or height of both the bedrock and ocean surface in what follows will always be shown relative to the initial ocean surface, and if not specified, the influence of the displacement of the center of mass of the Earth system (solid Earth, ocean, and land ice) is considered.

3.1. Bedrock Deformation for Circular Continents on the Equator

[18] We will first focus on the deformation of bedrock at equilibrium because it is closely related to the change of sea level. Figures 4 shows an example of the bedrock elevation at 5 Myr after the ice sheet load is applied, calculated using the method full_full defined previously. Results at 5 Myr are shown because the model has effectively reached equilibrium (see Figure 8) after this length of time and it is also close to the estimated lower bound (~4Myr) of the duration of a snowball Earth event [*Hoffman et al.*, 1998]. In this example, the radius of the ice sheet is 20 degrees and both its horizontal scale and total volume are close to those of the present Antarctic ice sheet. From Figure 4a, we can see that the bedrock under the ice sheet deforms most and the deformation is almost uniform except near the ice sheet margin. However, closer inspection of the depression (Figures 4b and 4c), demonstrates that the bedrock at the center of the ice sheet was depressed by ~532 m, and toward the edge of the ice sheet, the bedrock was depressed by an additional ~20 m. The slightly larger depression here compared to that at the center is due to the intrinsically high wave number loading near the edge and the assumed uniform thickness of the cylindrical ice sheet [*Wu and Peltier*, 1982]. For a realistic ice sheet, which is thick at the center and thins to zero

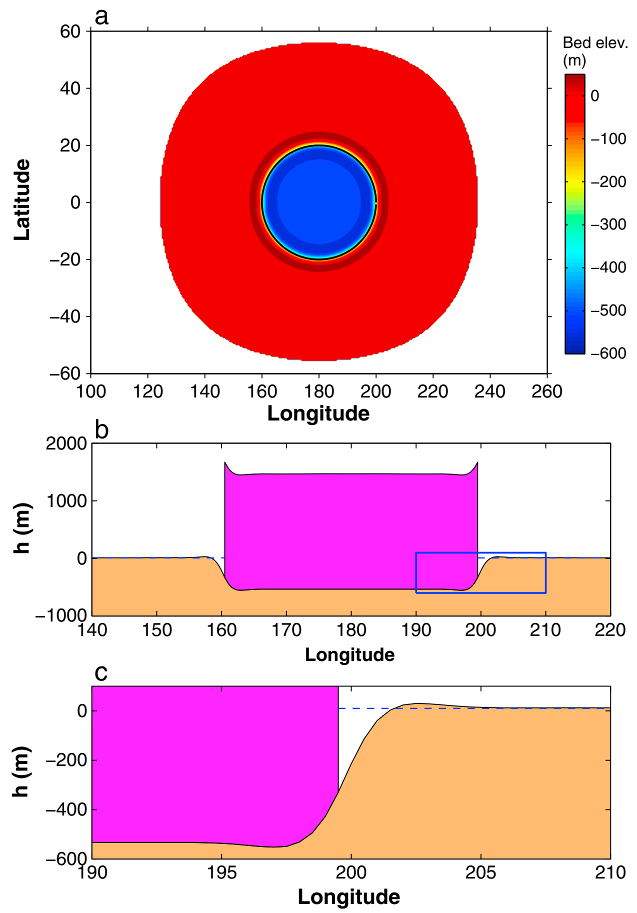

Figure 4. Bedrock elevation after 5 Myr calculated using method full_full. The ice sheet is cylindrical with radius of 20 degrees and height of 2000 m. (a) The top view of the elevation of the bedrock. The black circle around the blue region delineates the boundary of the ice sheet. (b) Vertical section along equator viewed from south. The ice sheet and the bedrock are shown as pink and orange, respectively. The original position of the bedrock surface is indicated by the dashed line. Note that only part of the continent is shown in Figure 4b. Figure 4c is the enlarged part of Figure 4b that is enclosed by the blue rectangle. The viscosity model is VM5a.

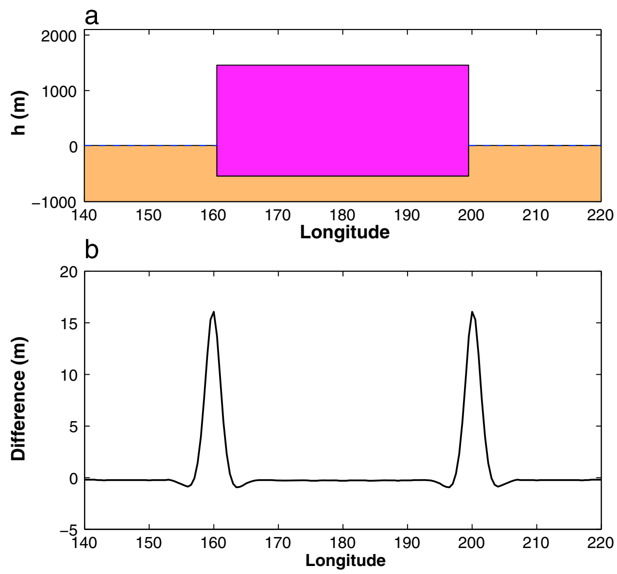

Figure 5. (a and b) Similar to Figure 4b except that these results are from methods expo_eust and full_eust, respectively. Figure 5b shows the difference between the bedrock deformations obtained using methods full_eust and full_full at equilibrium. The viscosity model is VM5a.

thickness at the margin, this deepening around the edge will not be observed. In Figure 4, a forebulge is clearly visible distant from the margin, the summit of which is higher even than the original elevation by ~30 m. The forebulge associated with ice loading has been recognized in the viscoelastic adjustment literature from very early in the development of the theory of this process [e.g., *Peltier*, 1974].

[19] Figure 5a shows the bedrock deformation for the same ice sheet but calculated using the method expo_eust. The most significant difference between Figure 5a and Figure 4b is that the depression of the bedrock in the former case is entirely uniform, a consequence of the fact that in the method expo_eust, only the local change of ice loading (or water loading if in the ocean basins) matters. Where there is no ice thickness change, the bedrock does not deform at all. With the density of 910 kg m^{-3} and 3300 kg m^{-3} assumed for the ice and bedrock, respectively, in this method, the depression should be 551.5 m uniformly. The bedrock deformation calculated by the method full_eust is almost the same as that predicted by full_full, demonstrating that little error in bedrock deformation will be caused by assuming that relative sea level changes eustatically for ice sheets of such size. This is true for the whole history of relaxation, with the error largest a few kyr after the ice sheets are emplaced and can reach ~6 m (negligible in relative terms) for the ice sheet with $r = 56$ degrees. The bedrock surface calculated by employing the full_eust methodology is generally slightly lower than that obtained using the full_full methodology (Figure 4b). This is because in the latter methodology, ocean water depth around the continent will be larger on account of the gravitational attraction by the ice sheet (as will be described in detail in what follows), and this water mass provides some buoyancy force to the bedrock over the continent. The two spikes around the ice edges in Figure 5b are due to an artificial effect as both methodologies assume that the cavity created by the depression of bedrock near the ice edges will be occupied by ocean water (which may exist only in these idealized experiments). Because the depth of these water-filled depressions in the full_full methodology will be deeper due to higher sea level there, the bedrock is locally depressed more than is the case in the predictions made using the full_eust method.

[20] Unlike the method expo_eust, the bedrock deformation using the other two methods is dependent of the size of the ice sheets, as demonstrated in Figure 6. Clearly, the depression of the land surface under the ice sheet is reduced as the radius of the ice sheet is increased, and the difference can be as large as ~30 m between the ice sheets of largest and smallest size tested here. The reason for this is that the larger ice sheets can "feel" the deeper part of the mantle which has higher density and they may be even large enough (e.g., $r = 40$ degrees) to deflect the core-mantle boundary which provides extra buoyancy force to the ice sheets. However, the error of ~20 m induced by employing method expo_eust is small compared to the uncertainties in determining the thickness of ice sheets, which may easily reach a few hundreds of meters due to the unconstrained or poorly constrained bedrock topography, ice sheet mass balance, and flow parameters, etc.

[21] This difference in bedrock deformation predicted using expo_eust and full_full for a more realistic circular ice sheet (Figure 3b), which is much thicker than 2000 m at the center, is ~55 m at both the center and the edge of the continent and smaller in between. Such small difference can be accommodated by an uncertainty in ice thickness of ~200 m (assuming density of ice and bedrock to be 910 and 3300 kg m^{-3} as in expo_eust method). Therefore, considering the large ice thickness in a snowball Earth event

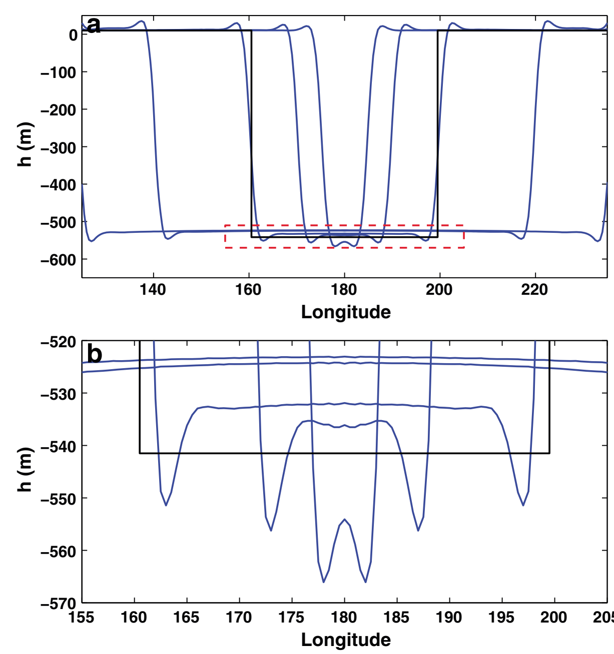

Figure 6. (a) The surface of the bedrock at 5 Myr for ice sheets of radius 5, 10, 20, 40, and 56 degrees predicted by method full_full. The solid black curve is that predicted by expo_eust for ice sheet of radius 20 degrees. The amount of depression does not change with radius for this method. (b) Enlargement of the part of Figure 6a that is enclosed by the red rectangle. The viscosity model is VM5a.

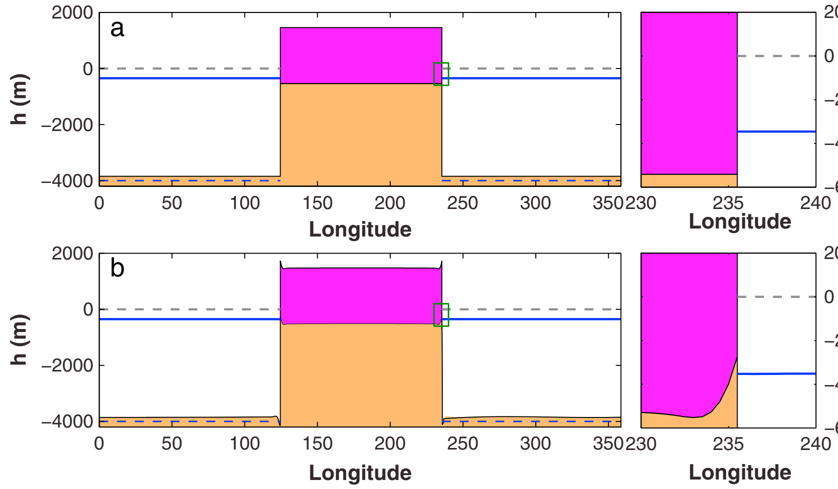

Figure 7. Vertical section along the equator similar to Figure 4b but here the ocean is also shown. This is the result for ice sheet radius $r = 56$ degrees at 5 Myr calculated using method (a) expo_eust and (b) full_full. The thick gray dashed lines and the blue solid lines are the initial and current ocean surface, respectively. The thin dashed lines indicate the initial ocean floor. The region around the continental boundary enclosed by the small rectangle is enlarged to the right.

(Figures 3c and 3d), we may infer that the snowball ice volume estimated using expo_eust method by [*Liu and Peltier*, 2010] is as accurate as that estimated using full_full method within the uncertainties of ice flow model.

3.2. Sea Level Changes at Equilibrium for Circular Continents on the Equator

[22] At equilibrium, both the ocean surface and ocean floor are very "flat" for all three methods, but only results from expo_eust and full_full are shown in Figure 7. The ice sheet applied to the surface of the supercontinent of radius 56 degrees is similar to what one expects to have been characteristic of a snowball Earth state in which the ice volume was equivalent to a 501 m (eustatic) depression of sea level. The global mean ocean surface is found to be ~350 m below the reference surface (the initial ocean surface) for all three methods, smaller than 501 m (ice volume equivalent sea level change) because the ocean bottom is elevated by the rebound of the ocean floor due to the removal of the water load. The elevation of ocean floor obtained by the full_full method is slightly smaller (~6 m) than that obtained by application of the expo_eust method due to the influence of higher deeper mantle density and the deflection of the core-mantle boundary as have been described above; therefore, the ocean surface is correspondingly slightly lower.

[23] Although the ocean surface height is very similar for all three methods, the relative sea level at the coastline of the continent (i.e., freeboard) can be very different due to the formation of the forebulge in methods full_eust and full_full (Figure 4). Without this bulge, the ocean surface will be higher than the bedrock surface by ~190 m at the edge of continent (Figure 7a). While with the bulge, the ocean surface is lower than the bedrock surface by ~77 m, recording a sedimentologically observable sea level fall at the edge of continent (Figure 7b). The full_eust predicts an even larger sea level fall (by ~280 m) than the full_full method due to the deep depression of the sea floor near the edge of the continent (see Figure 7b since the full_eust method predicts very similar bedrock deformation) since it is assumed that the ocean depth is uniform in the former method (not shown).

[24] The equilibrium height of the forebulge is much more sensitive to the thickness of effectively elastic lithosphere than to the viscosity of the other part of the Earth. When the viscosity models VM5aL90 and VM5aL120, in which the thickness of lithosphere is 90 km and 120 km, are employed, the edge of the continent is higher than that obtained for model VM5a (Figure 7b) by ~12 m and ~22 m, respectively. If the thickness of lithosphere is the same but the viscosity of the deeper part of the mantle is increased, e.g., lm4um1L90 vs. lm100um1L90, the height of the forebulge differs by less than 5 m at 5 Myr. Different viscosity models therefore have little impact on the equilibrium freeboard values predicted for a snowball Earth event, but they do influence the timescale over which these deformations and changes were obtained as will be discussed in the next section.

3.3. Evolution of Bedrock and Sea Level With Time for Ice Sheet-Loaded Circular Continents

[25] Because the ice sheets have been assumed to be emplaced instantaneously on the continents in all of the previously discussed calculations, the results cannot be employed to obtain realistic sea level histories during the snowball Earth formation process. However, it is still useful and important to understand the timescale required for the bedrock and sea level system to reach equilibrium for ice sheets of different scale. Although some insight might be gained from theoretical analyses such as those of *Peltier* [1974, 1976] regarding the evolution of the displacement of bedrock with time, for ice sheet loads of different horizontal scale, the characteristic timescale for the adjustment of the ice-Earth-ocean coupled system has never been fully investigated.

[26] Figure 8 shows the time series of the bedrock elevation at the center of the continent for methods expo_eust and full_full. The relatively short time series (100 kyr) in Figure 8a all share the same quasi-exponential relaxation characteristics in that they all decrease rapidly at the

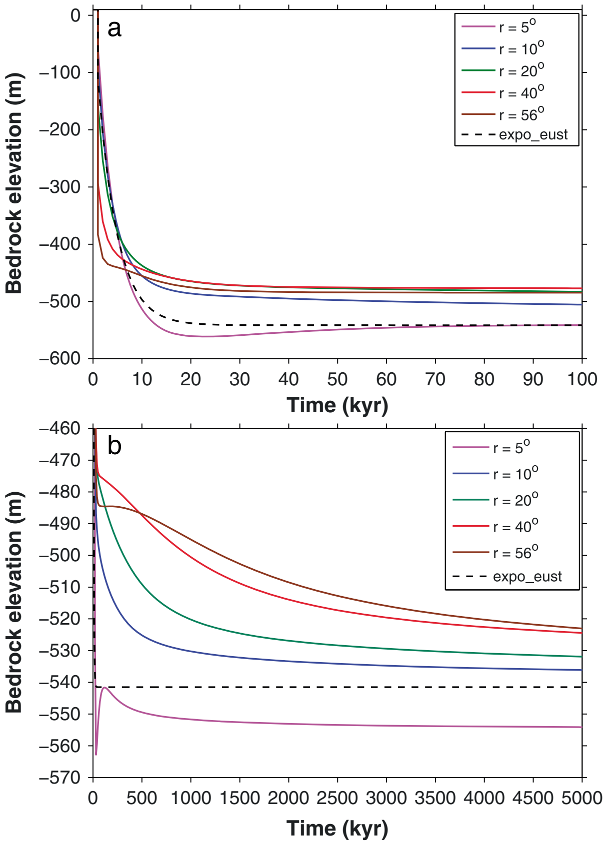

Figure 8. Time series of the bedrock elevation at the center of the continent obtained for viscosity model VM5a. The solid curves are calculated by method full_full for different size of ice sheets, and the dashed curve, which is the same for ice sheets of all radii, by method expo_eust. Figures 8a and 8b are same except for that the length of the time series in the latter is longer and the scale of the *y* axis is larger.

beginning, a phase that is followed by a slower approach to an eventual equilibrium. However, only evolution of the bedrock deformation for the smallest ice sheet can be relatively well represented by the exponential decay form with an *e*-folding time of 4 kyr. The longer time series (5 Myr) in Figure 8b are all characterized by an initial sharp drop in elevation within the first ~50 kyr and then followed by slow quasi-exponential relaxation with much larger *e*-folding time than 4 kyr. This is the case even for the smallest size of the ice sheet, but the total magnitude of adjustment achieved by this slow relaxation is only a small portion (~10%) of the total adjustment achieved since the beginning for most of the viscosity models tested (see Figures 9 and 10 below). The stretching of the adjustment time as a function of time is due to higher viscosity in the lower mantle (Figure 2b). The nonmonotonic increase of the bedrock deformation seen for the smallest ice sheet (pink curve in Figure 8) is due to relatively complex behavior of the lithosphere (not shown). When the thickness of the lithosphere is increased to 90 km or more, such behavior disappears (see Figure 9).

[27] Figure 9a demonstrates that different viscosity models have little influence on the time evolution of the bedrock deformation for small ice sheets except when the lithosphere is thin as described above. But for large ice sheets, the increase of lower mantle viscosity (which is not well constrained) can increase the timescale of the solid Earth isostatic adjustment significantly (Figure 9b). However, after a few hundred thousand years following emplacement of the ice sheets on the continent, even the model with highest lower-mantle viscosity has very nearly achieved its eventual equilibrium state. This is more clearly demonstrated in Figure 10.

[28] Figure 10 not only shows that the isostatic adjustment of the solid Earth has almost achieved equilibrium after 500 kyr, but also that the magnitude of the instantaneous elastic deformation increases dramatically with the size of the ice sheets. Taking the point at the center of the ice sheet as an example, for the ice sheet of radius $r = 5$ degrees, the instantaneous elastic deformation is only ~55 m, approximately 10% of the total deformation at equilibrium. For the ice sheet of radius $r = 56$ degrees, however, the elastic deformation is 383 m, almost 75% of the total deformation at equilibrium. Clearly, such an effect cannot be captured by the expo_eust method, but it does provide very good estimate of the bedrock deformation for the equilibrium state (Figures 6 and 8).

[29] The analyses above demonstrate that showing results for VM5a is sufficient for the purpose of this paper since the adjustment of relative sea level itself is also determined by the isostatic adjustment of the solid Earth as will be shown by Figure 11. However, it will be demonstrated in *Liu and*

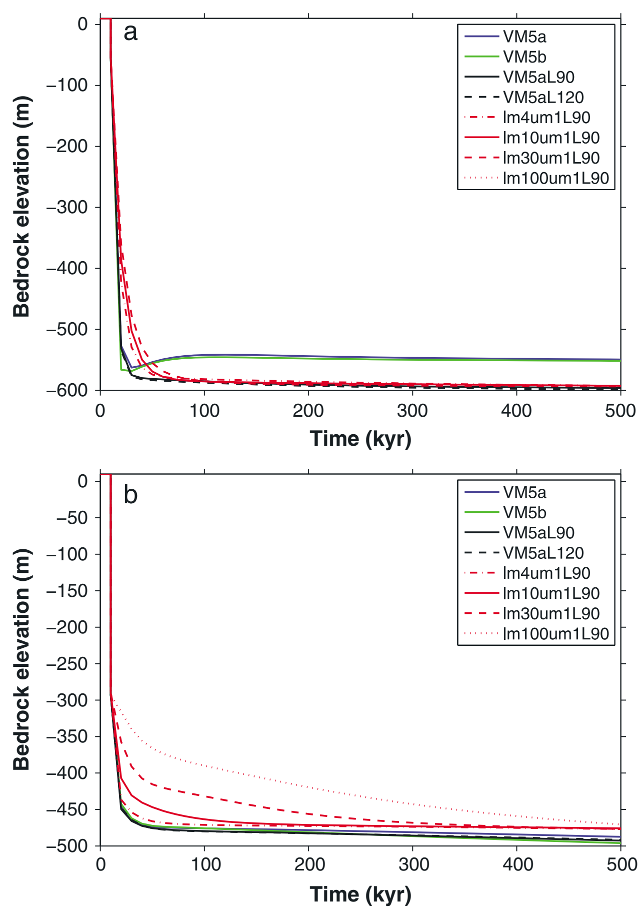

Figure 9. Similar to Figure 8 but here the results for all the viscosity models are shown for ice sheet or radius (a) 5 degrees and (b) 40 degrees.

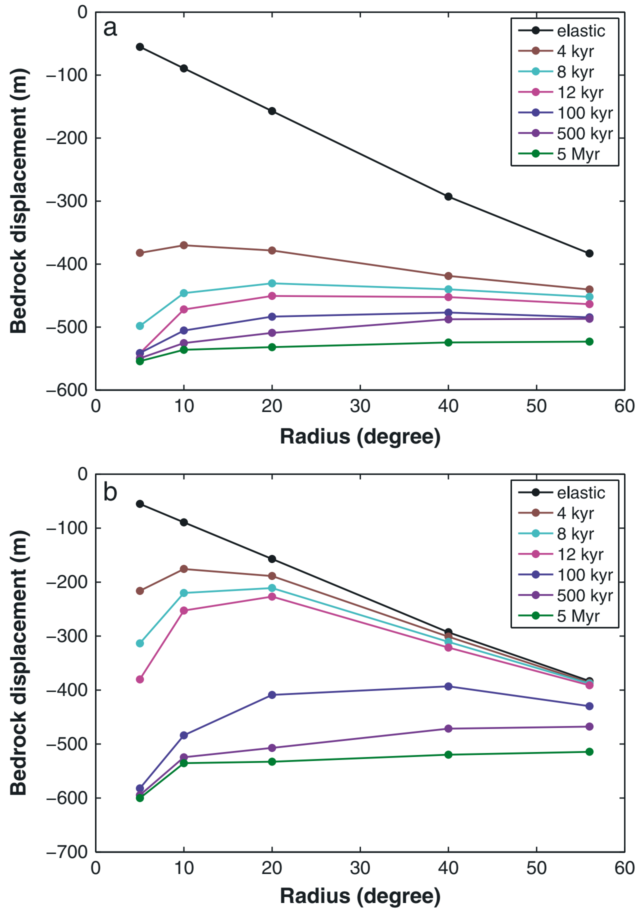

Figure 10. Instantaneous elastic deformation of the bedrock at the instant the ice sheet load is applied compared to the total deformation achieved at time 4 kyr, 8 kyr, 12 kyr, 100 kyr, 500 kyr, and 5 Myr, respectively, at the center of the ice sheets. These results have been computed using the full_full methodology for viscosity models (a) VM5a and (b) lm100um1L90.

Peltier [2013] that different viscosity models have very significant influence on the rotation of the Earth due to snowball Earth formation.

[30] At the instant or very shortly after the ice sheet is emplaced on the continent, the relative sea level is highly nonuniform and monotonically increases from the center of the ocean basin toward the edge of the continent due to the gravitational attraction of the ice sheet (Figure 11a). However, the absolute ocean surface height relative to its original undisturbed position falls toward the ice sheet due to the decreased uplift of the ocean floor with increasing proximity to the glaciated continent. In the very near field of the continental ice cover, however, the surface is once more warped upward somewhat due to the very large gravitational attraction by the ice sheet which in this region dominates the depression of the ocean floor (Figure 11a). As the solid Earth adjusts, the relative sea level becomes almost uniform except very near the edge of the continent, and the ocean surface becomes almost flat everywhere (Figure 11b). The timescale at which the relative sea level (or water depth) adjusts is shown in Figure 11c and is demonstrated to be the same as that of the bedrock (see Figure 8a).

3.4. Realistic Neoproterozoic Continental Configurations

[31] For the 720 Ma continental configuration shown on Figure 3, the total land area is only ~110×10^6 km^2, much smaller than that of the present continental configuration (~150×10^6 km^2) and even less than that of the 570 Ma configuration (~130×10^6 km^2), for the reasons discussed in *Liu and Peltier* [2010]. We consider this 570 Ma configuration to be a reasonable approximation to the Marinoan snowball configuration that was obtained about 50 Myr earlier at 620 Ma. Moreover, the individual continental fragments were not well connected to each other (even after the slight modifications applied in *Liu and Peltier* [2010]) for the 720 Ma continental configuration, which led to the result that the ice sheets in the snowball state would have consisted of several smaller ice domes (Figure 3c). Each of the ice domes had a smaller thickness compared to that of the single dome for the 570 Ma continental configuration (Figure 3d). Therefore, the ice volume for the 720 Ma continental configuration in the snowball state was found to be only ~750 m in eustatic sea level equivalent terms. Given that the average uplift of the ocean floor in this circumstance is ~225 m due to this sea level change, the mean ocean surface lowering expected would be ~525 m. It is found that there is virtually no difference in either the ice area or ice volume between the soft and hard snowball states for this continental configuration since in either of these states land ice would have covered all of the continental surfaces.

[32] For the 570 Ma continental configuration, however, there were noticeable differences in ice coverage between the soft and hard snowball states. In a soft snowball state, two small continental fragments, which were significantly displaced from the major continents, were predicted to be ice free [e.g., see *Peltier et al.*, 2007, Figure 5]. In a hard snowball state, on the contrary, all the continental fragments would be covered by ice (Figure 3d). The ice sheet coupled energy balance model predictions of *Liu and Peltier* [2010] for continental ice volume are ~1013 m and ~ 1047 m eustatic sea level equivalent, for the soft and the hard snowball states, respectively, either of which is much larger than that for the 720 Ma continental configuration discussed previously. Again, considering the average uplift of the ocean floor by ~310 m, the actual mean ocean surface lowering would be ~710 m and ~735 m, respectively. Although the ice volume is therefore only slightly different between the two snowball states, it will be shown that the consequent freeboard obtained for the continental fragments that were distant from the major land masses is significantly different.

[33] The analyses above have demonstrated that the equilibrium ocean surface remains almost flat after an ice sheet of any size is emplaced on the continent; therefore, the variation of freeboard along coastlines of continents is determined by the spatial variation of ice sheet thickness and the corresponding bedrock deformation. Figure 12 shows the bedrock elevation over the continents for both 720 Ma and 570 Ma continental configurations, as well as freeboard values along the coastlines at 5 Myr. It can be seen that the freeboard values are highly variable and are generally much smaller than the ocean surface lowering (contours over

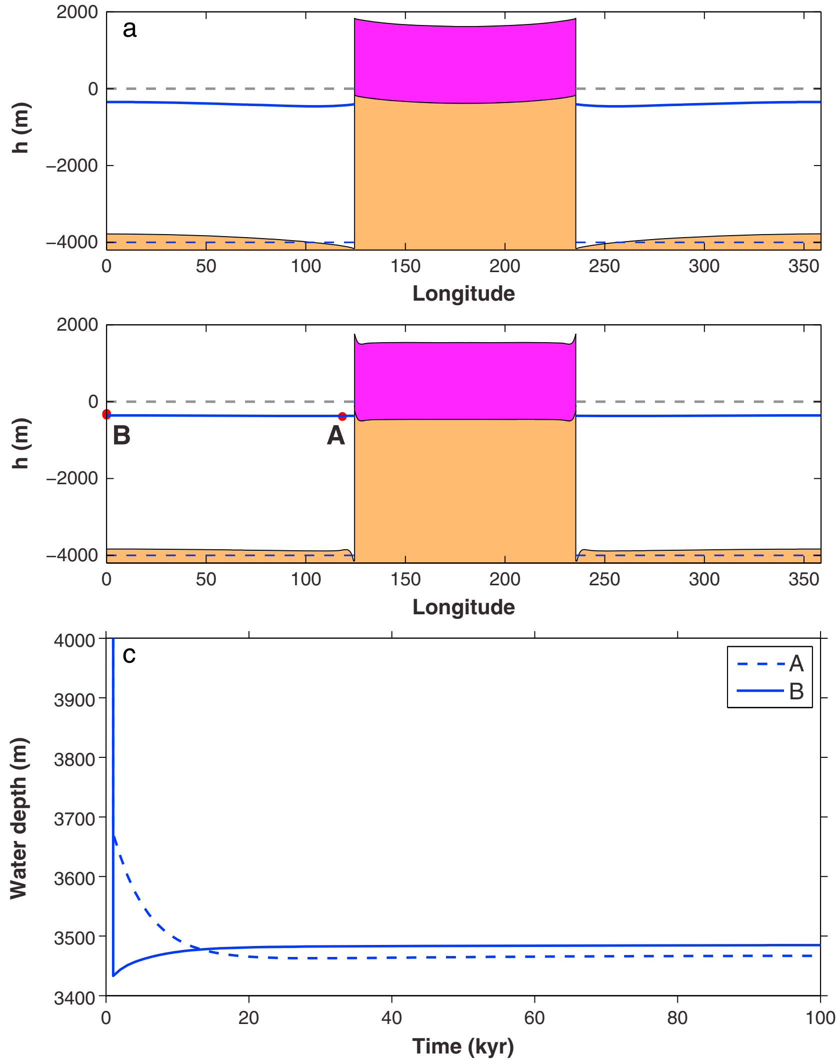

Figure 11. Bedrock elevation and sea surface along equator at (a) the instant ice sheet was loaded on the continent and (b) 12 kyr after the ice sheet was loaded calculated by method full_full. (c) Time series of water depth (relative sea level) at two points "A" and "B" as indicated in Figure 11b.

the ocean in Figure 12). Since the occurrence of smaller freeboard values than the ocean surface lowering is due to depression of the land surface by ice sheets, it is generally the case that the freeboard values are the smallest where the ice thickness is largest.

[34] Histograms of freeboard values along the coastline for both 720 Ma and 570 Ma continental configurations are shown in Figure 13, where all the coastal points on a 0.5° × 0.5° gridded map are counted. The freeboard values for the 720 Ma continental configuration (Figure 13a) can be reasonably approximated by a Gaussian distribution with mean of ~400 m and standard deviation (σ) of 60 m. Therefore, a 2σ (95%) range of plausible freeboard values would be 280 – 520 m. The freeboard values for the 570 Ma continental configuration (Figures 13b and 13c) are probably better described by a Laplace distribution with nonzero skewness peaking at ~620 m (Figure 13b) and ~600 m (Figure 13c) for the hard snowball and soft snowball Earth, respectively. The 95% confidence interval of freeboard values for the hard snowball case (Figure 13b) is approximately 530 – 890 m. For the soft snowball case (Figure 13c), this interval would be similar if the coastlines of the two small continental fragments which are not covered by land ice were not considered.

[35] The influence of the thickness of the elastic lithosphere on the freeboard values is such that the histogram as a whole shifts toward smaller values with larger thickness, but by a small amount (~10 – 20 m, Figure 13). This is opposite to what is found for the cylindrical ice sheet in section 3.2. A larger underestimation of the freeboard values (by > 100 m, Figure 13) occurs if the method expo_eust is employed. This is mainly due to the inability of this method to predict the formation of a proglacial forebulges (Figure 7) and partly due to the overestimation of the uplift of the ocean floor (by ~10 m).

[36] For the 570 Ma continental configuration, the freeboard predicted for the ice sheets of a soft snowball state (Figure 12c) was generally smaller than that obtained for a hard snowball state (Figure 12b) by ~25 m. However, along the coastlines of two small continental fragments, which were relatively far from the major land masses and were

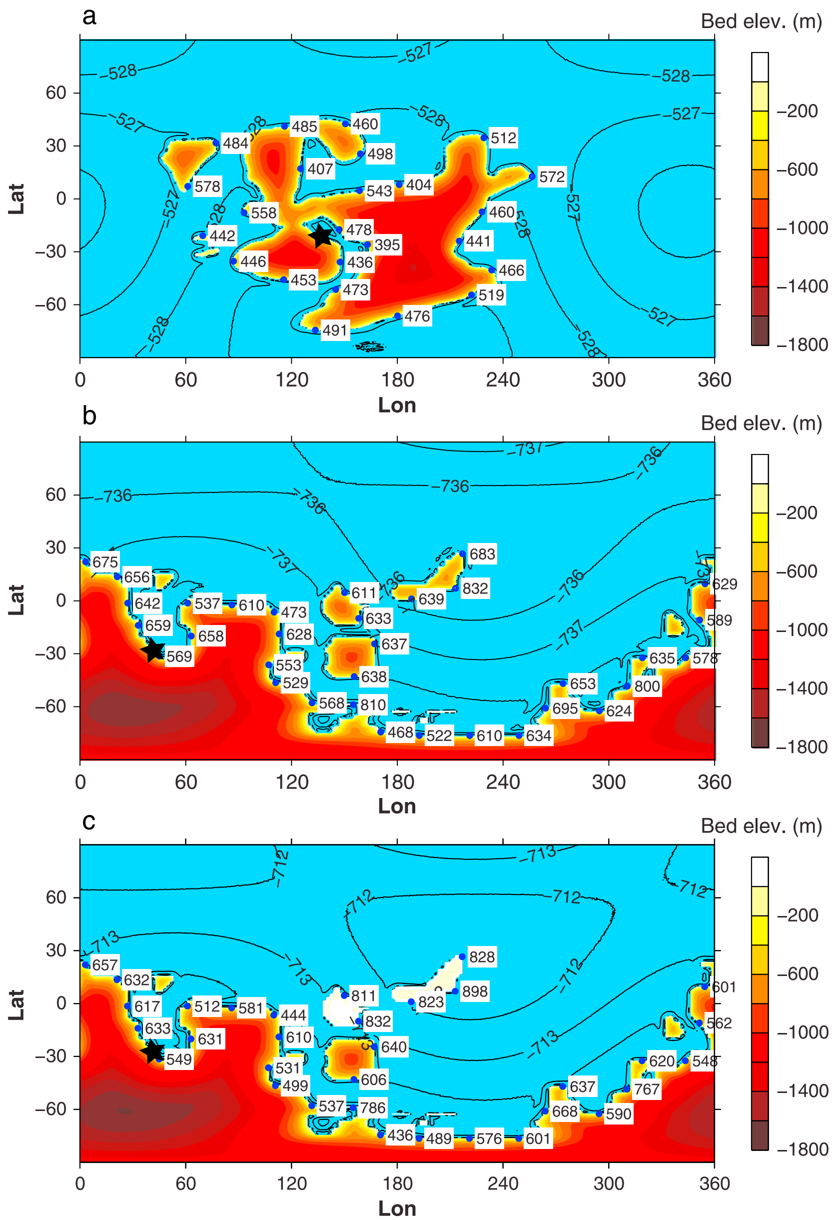

Figure 12. Ocean surface lowering (contours), bedrock elevation (filled contours) relative to the initial ocean surface, and freeboard values (numbers in white boxes) for (a) 720 Ma, (b) 570 Ma continental configuration with ice sheets associated with a hard snowball state, and (c) 570 Ma continental configuration but with ice sheets associated with a soft snowball state. The freeboard values are corresponding to locations indicated by the blue dots. The location of Otavi Group is crudely estimated and indicated by the black pentagram.

not covered by ice in the soft snowball state, the freeboard was actually much larger in the soft snowball state. This difference in freeboard is clearly due to the fact that the continents (including the coastal regions) were depressed by a few hundred meters when they were covered by ice in a hard snowball state.

[37] We expect that the theoretical calculations presented herein provide an upper bound estimate of the freeboard, which could be inferred on the basis of evidence derived from relevant geological sections during snowball Earth events. These estimates are upper bounds primarily because no preexisting continental topography was included in the model of ice sheet formation, and also to some degree because bedrock depression was overestimated in the ice sheet model (e.g., Figure 14). However, comparison of the theoretical results presented here with the possible observations cannot be expected to be as reliable as are those for the ice sheets of the Late Quaternary ice age. For this much more recent era, there exists a wide range of data that can be brought to bear to constrain ice sheet mass and therefore the change in ocean volume. These data include direct measurements of the rise of sea level since the last maximum of glaciation based upon coral-derived records from the tropics [e.g., *Peltier and Fairbanks*, 2006], oxygen isotope-derived records from radio-carbon dated deep sea sedimentary cores [e.g., *Shackleton et al.*, 1990], and records of land

uplift (post glacial rebound) from the once ice-covered regions of the continents [e.g., *Peltier*, 1998, 2007]. For the Neoproterozoic interval of time, none of these methods of inference are available. All we have are sedimentological records on the basis of which the freeboard in the deposition zones may be estimated.

[38] The only evidence that has been interpreted as indicative of a large (~500 m) sea level fall is that from the Otavi Group sediments of Northern Namibia [*Hoffman et al.*, 2007] for the Marinoan glaciation. The interpretation of this sedimentary section relies upon the assumption of specific mechanisms for the formation of the sediments of different facies and at different heights in the section. *Hoffman et al.* [2007] believe their estimation of sea level fall to be conservative, i.e., that the actual sea level fall would have been (perhaps significantly) larger. If their estimate is assumed to be reasonably accurate, then the freeboard estimates delivered by our calculations for the 570 Ma continental configurations are broadly consistent with the estimate derived from the Otavi section (see the very crudely estimated location of the Otavi Group on the freeboard map associated with the 570 Ma glaciation in Figure 12).

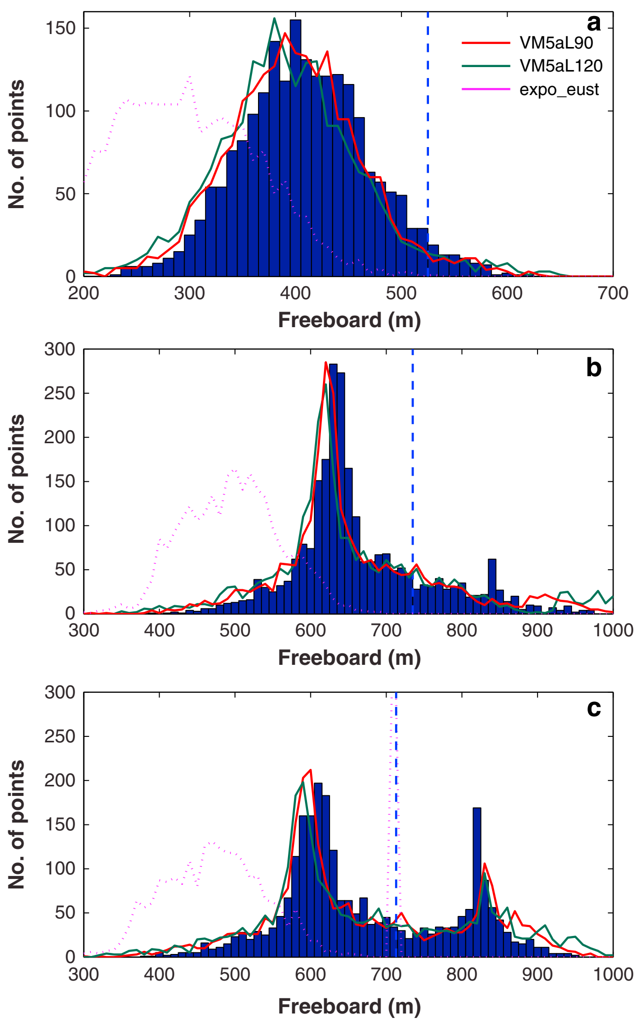

Figure 13. Histogram of the freeboard values in Figure 12 at interval of 10 m for all the coastal points, which are identified on a 0.5° × 0.5° gridded map. The blue dashed lines indicate the mean ocean surface lowering. Red and green curves are the envelope of histogram obtained for two viscosity models with thicker elastic lithosphere.

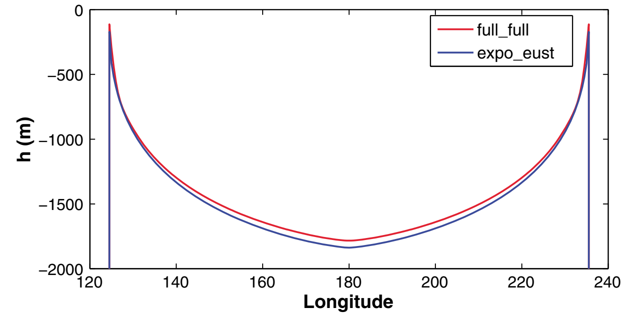

Figure 14. Bedrock elevation at 5 Myr under the realistic circular ice sheet (Figure 3b) predicted by methods full_full and expo_eust for viscosity model VM5a.

[39] However, as shown explicitly in Figures 12 and 13, the freeboard along the coastlines would be expected to have been highly variable. We would have to have much more accurate paleo-locations of such observations in order to compare them with the theoretical calculations. This might then be employed to constrain total ice volume. Unfortunately, even the reconstructions of paleo-continental configurations for the period of interest are still controversial, and results differ significantly among different studies from which results are available in the recent literature (consider, for example, the difference between the reconstructions in *Trindade and Macouin* [2007], *Li et al.* [2008], and *Evans* [2009]).

[40] Moreover, acceptably accurate detailed paleo-topography is also required for inferring sea level change from either geological sections or theoretical calculations. For example, the evidence of sea level change in Northern Namibia [*Hoffman et al.*, 2007] was obtained by assuming the initial sea level was close to the top of the continental slope. If the initial (prior to glaciation) sea level was actually higher than this, then the sea level change would have been underestimated. Further problems clearly exist if the action of local tectonic processes is considered, especially if the snowball formation process were sufficiently slow.

4. Conclusions

[41] The sea level changes to be expected during a snowball Earth event have been analyzed herein by employing the Sea Level Equation (SLE) formalism and approximations to it (e.g., see *Peltier* [1998], *Peltier and Luthcke* [2009], and *Peltier et al.* [2012] for recent reviews) for models of different complexity. The first methodology employed, referred to as expo_eust herein, computed bedrock deformation according to a simple exponential return to equilibrium methodology with a fixed exponential relaxation time of 4 kyr and described the change of sea level eustatically. The second methodology, denoted full_eust, improves upon the first by calculating the bedrock deformation using a highly accurate viscoelastic model of the isostatic adjustment process based upon the viscoelastic Green function methodology of *Peltier* [1974, 1976] (see *Peltier* [1998] and *Peltier et al.* [2012] for recent reviews).

The third methodology, denoted full_full, improves further upon the accuracy of these calculations by determining sea level change using the complete gravitationally self-consistent Sea Level Equation methodology. All three methods have been widely employed in past studies of ice sheet evolution and sea level change during the Late Quaternary period [e.g., see *Tarasov and Peltier*, 2002; *Tarasov et al*., 2012]. Our interest in the present paper has been in the nature of the ice-Earth-ocean interaction process that is expected to be characteristic of the ice sheets that are believed to have grown on the supercontinents that existed during the Neoproterozoic period of Earth history, when the ice sheets are believed to have been as much as an order of magnitude greater in total volume than those of the Late Quaternary. It is shown that the simplest of the above methods is able to provide a fairly accurate description of the equilibrium bedrock deformation over all scales, therefore is approximately accurate when coupled to an ice sheet model to estimate the total ice volume during snowball Earth events, but may not be suitable if short timescale ice sheet dynamics are to be considered. It is definitely not appropriate to use this method to estimate the freeboard values during snowball Earth events.

[42] A comparison of the methods expo_eust and full_full demonstrated that the isostatic adjustment of the bedrock became slower as the horizontal scale of the ice sheets and the viscosity of the mantle increased, but even for an ice sheet of expected snowball scale and the highest viscosity model tested, the Earth system reaches near equilibrium after 500 kyr (Figure 10). Therefore, for snowball Earth events, which allegedly lasted for multiple million years [*Hoffman et al*., 1998], considering the freeboard values only of the equilibrium state is entirely appropriate.

[43] For the simplified experiments analyzed herein, in which the ice sheets were emplaced upon the continents instantaneously, both the absolute and relative magnitudes of the initial elastic deformation of the bedrock were found to increase significantly with horizontal scale of the ice sheets. This elastic deformation comprised only ~10% of the total adjustment of the bedrock when the (cylindrical) ice sheets were assumed to be small in radius, while for snowball scale ice sheets, the magnitude of this elastic deformation approached ~75% of the equilibrium viscoelastic deformation (Figure 10).

[44] In the equilibrium state, the ocean surface is almost flat, but the freeboard values are found to vary significantly due to differential bedrock depressions in the coastal region. For the 720 Ma continental configuration, the ice sheets in the snowball state (either hard or soft) had a volume of ~750 m sea level equivalent, but the ocean surface relative to its original position was lowered by only ~525 m due to the influence of the uplifted ocean floor. The freeboard values were generally within 280 m – 520 m (Figures 12a and 13a). For the 570 Ma continental configuration, the ice sheet volume was ~1013 m sea level equivalent in a soft snowball state and ~1047 m sea level equivalent in a hard snowball state. Also, due to ocean floor uplift, the ocean surface was lowered by only ~ 720 m. The freeboard values were in the range of 530 – 890 m. However, for two small continental fragments, which were distant from the major land masses and were not covered by ice in the soft snowball state, the freeboard was predicted to be much larger in a soft snowball state (~820 m, Figure 12c) than that in a hard snowball state (~640 m, Figure 11b) due to the higher elevation of the continents. Different viscosity models have no significant influence on the equilibrium freeboard values described above. The calculated freeboards delivered by the analyses discussed herein are broadly consistent with the only available inference, ~500 m from Northern Namibia as discussed by *Hoffman et al*. [2007].

[45] **Acknowledgments.** The required computations were performed on the SciNet facility at the University of Toronto, which is a component of the Compute Canada HPC platform. Additional support for the work reported herein has been provided by NSERC Discovery Grant A9627 to WRP. We are indebted to Rosemarie Drummond for technical assistance in the sea level analyses reported herein. We thank B. Steinberger for his constructive commentary.

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